Vol. XXXIX. JUNE, 1898.

AMERICAN SOCIETY OF CIVIL ENGINEERS.

INSTITUTED 1852.

TRANSACTIONS.

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No. 819.

PRESSURES RESULTING FROM CHANGES OF VELOCITY OF WATER IN PIPES.

By J. P. FRIZELL, M. Am. Soc. C. E.
PRESENTED OCTOBER 6TH, 1897.

WITH DISCUSSION.

The pressures that may be generated in pipe lines by the sudden closing of valves or the sudden starting of pumps have long occupied the attention of hydraulic engineers, and some attempts have been made, both experimentally and analytically, toward a better understanding of the subject.

Edmund B. Weston, M. Am. Soc. C. E., presented to this Society, in 1884, an account of some experiments made upon water pipes at Providence, R. I., with the view of ascertaining the maximum pressure incident to the sudden stoppage of water in a pipe.* The pipes were short and of small diameter. The pressures were recorded by a style which drew a straight mark upon stationary paper, and the diagram took no account of the oscillations which succeeded the main shock incident to the stoppage of the current. The valve which caused the stoppage occupied an appreciable time in closing. The results, though praiseworthy as an attempt at the elucidation of an important subject, are of limited practical value.

* See *Transactions*, Vol. xiv, p. 238.

A somewhat higher degree of value can be accorded the work of Professor R. C. Carpenter, of Sibley College, Cornell.* In this case the style recorded the pressures on paper moving isochronously, and represented the oscillations succeeding the main shock. The pipes used were of small diameter and only a little over 50 ft. in length; being, according to what follows, much too short to fully develop the pulsation incident to closing the valve, especially when the closing is far from being instantaneous. Professor Carpenter adds some theoretical discussion, which displays a comprehension of the elements of the question.

Professor I. P. Church has published a paper† aiming to treat the whole question in a rigorously exact manner, determining the pressure and velocity in the pipe for different positions of the valve while closing. The results are complex and intricate. The writer makes

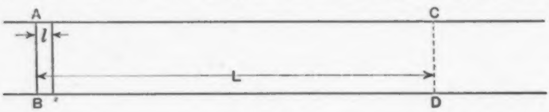

FIG. 1.

no practical application of them, and regrets his inability to pursue the subject further.

Recently this subject has been brought into renewed prominence from its connection with the regulation of water-wheels drawing through long penstocks, and from the further fact that the requirements of electric transmission call for greatly increased exactness in the regulation of wheels. Feeling that anything tending to a better understanding of the subject would be acceptable under present conditions the author is emboldened to offer the following views:

Assume a long pipe filled with water (Fig. 1). Imagine at $A\ B$ a piston to start forward. When it has moved the distance l, it has set the water in motion as far as $C\ D$.

Let r = radius of pipe in feet.

Let W = weight of a cubic foot of water in pounds.

Let m = modulus of elasticity of water = 20 000 atmospheres, say 294 000 lbs. per square inch.

Let M = modulus of elasticity of metal of pipe, in pounds per square inch.

Let T = thickness of pipe metal, in inches.

* Published in the *Transactions* of the Am. Soc. M. E., Vol. xv, p. 510.
† See the *Journal* of the Franklin Institute, April and May, 1890.

Let t = time occupied by the piston in moving the distance l, in seconds.

Let f = force exerted upon the water by the piston, in pounds per square inch.

When the piston has moved a distance l, although it has only moved the center of gravity of the mass L, a distance $\frac{1}{2} l$, it has done work in compressing the water and distending the pipe equivalent to moving the mass a distance l, and imparting to it a velocity $\frac{l}{t}$. This will appear on reflecting that, if the compression and distension could be released without any further obstruction to movement, the mass of water would have the motion stated.

The increase in the radius of the pipe from the pressure f is $\frac{r^2 f}{M T} = \varDelta r$.

Increase in cross-section $= 2 \pi r \varDelta r = 2 \pi \frac{r^3 f}{M T}$.

Increase of volume $= L f \frac{2 \pi r^3}{M T}$.

Traverse of piston due to distension of pipe $= L f \frac{2 r}{M T}$ ....(1)

Traverse of piston due to compression of water $= \frac{L f}{m}$ ....(2)

Total traverse of piston in time $t = l = L f \left(\frac{2 r}{M T} + \frac{1}{m}\right) \therefore$

velocity $= \frac{L f}{t}\left(\frac{2 r m + M T}{m M T}\right)$ ....(3)

The total weight of water set in motion is $\pi r^2 L w$.

The force acting to impart motion is $144 \pi r^2 f$ pounds.

Gravity acting freely would impart to this mass a velocity of $g t$ feet per second in the time t. Therefore there results the proportion

$$\frac{l}{t} : g t = 144 \pi r^2 f : \pi r^2 L W$$

whence

$$L = 144 f \frac{g t^2}{l W} = 144 \frac{g t^2}{W} \frac{1}{L} \frac{m M T}{2 r m + M T} \quad \text{....................(4)}$$

and the velocity of the pulsation in the pipe is

$$\frac{L}{t} = \sqrt{144 \frac{g}{W} \frac{m M T}{2 r m + M T}} \quad \text{..................................(5)}$$

Assuming a 60-inch pipe of steel $\frac{1}{2}$ in. thick, $T = \frac{1}{2}$, $M = 30\,000\,000$, there results $v = 4\,272$ ft. per second.

If the distension of the pipe is neglected, by assuming an infinite value for M, (5) may be put in the form.

$$\frac{L}{t} = \sqrt{144 \frac{g}{W} \frac{m}{\frac{2 r m}{M T} + 1}}.$$

The assumption that no distension of the pipe takes place is equivalent to making M equal to infinity. On this assumption the above equation becomes

$$\frac{L}{t} = \sqrt{144 \frac{g}{W} m} = 4\,672 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(6)};$$

being substantially the velocity of sound in water, which is usually taken at about 4 700 ft. per second.

Instead of the supposition of a piston suddenly starting forward, imagine water, moving with a uniform velocity v, to be suddenly arrested by the closing of a gate. Then the force f is the force acting against the gate and walls of the pipe, in excess of the static pressure, and is found thus: Let X be the length of the pipe, and for simplicity suppose X to exceed $\frac{L}{t}$ or the value of L when $t = 1$. In one second after closing the gate, the length L is condensed into the length $L - v$, and the diminution of volume by compression is represented by $\frac{v}{L} \frac{M T}{2 r m + M T}$ and

$$f = \frac{v}{L} \frac{M T m}{2 r m + M T} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(7)}$$

Taking $v = 4$ ft., the other symbols as before, then $L = 4\,272$

$$f = \frac{4}{4\,272} \frac{7\,500\,000}{5 \times 294\,000 + 7\,500\,000} \times 294\,000$$

$= 230$ lbs. per square inch.

It will appear from (6) that f is theoretically independent of the length of the column of water in motion, *i. e.*, the length of the pipe. This, however, involves the assumption that the stoppage is absolutely instantaneous. Under practical conditions, in which the stoppage occupies an appreciable time, the force developed is not independent of the length. For the purpose of experiment, a form of valve may be adopted, causing an instantaneous stoppage, though such valves are ordinarily avoided. Assuming such a valve to be used, upon closure, the full pressure is instantly developed upon the valve. The section in which the water is coming to rest moves up stream with the velocity indicated by equation (5). The amplitude of this movement is only limited by the length of the pipe, above the valve, either to the reservoir or to the larger pipe from which it branches.

Neglecting the elasticity of the pipe, the time occupied by its contents in coming to rest is $\frac{X}{4672} = t$. The water has continued to flow into the pipe with the undiminished velocity v for t seconds after the closure. The compression of the water in the pipe is represented by $\frac{v\,t}{X}$, and the force exerted on the interior by $m\frac{v\,t}{X}$ pounds per square inch.

The pressure on the valve, however, is not released when the entire contents have come to rest. The water, now, through the entire length X is in a state of compression far above the normal, and a release and reversed motion takes place. This commences at the origin of the pipe and moves toward the valve with the velocity of equation (5). The time therefore between the stoppage and the release of pressure will be $2\,t$. If the normal static pressure in the pipe is f or more, then the pressure at the valve will fall as much below the normal at the release as it rose above the same at the closure, except in so far as the movement is affected by fluid friction, and a series of pulsations with the cyclic interval $2\,t$ will run through the pipe till the movement dies out.

If the pipe branches from another of larger diameter, then, when the pulsation of closure reaches the larger pipe, it will continue through the same with diminished intensity, the pressure being proportional to the velocity. A pulsation of release will return through the smaller pipe, partially releasing the pressure on the valve. The pulsation of closure will reach the head of the larger pipe, and a pulsation of release will return through both pipes, etc. A complex system of pulsations will ensue, depending on the relative lengths of the pipes. Very different is the case when the arrest of motion occupies an appreciable time. The closing may be supposed to take place by a great number of small steps, each of which may be regarded as instantaneous. Each step occasions a certain diminution of velocity and is accompanied by an increment of pressure. Each increment runs to the head of the pipe, is discharged and returns in the same manner and with the same velocity as before. No marked increase of pressure takes place at the head of the pipe during the closing. Every increment of pressure originating at the valve remains in force there till the pulsation has run to the head of the pipe and back to the valve.

It is now possible to perceive clearly the defects of the experiments made by Mr. Weston at Providence, and by Professor Carpenter at

Cornell. In the former the valve was thought to occupy 0.15 second in closing, an interval sufficient for the pulsation to run 600 or 700 ft. The pulsation lasted as long as the closing, and its commencement had extended far beyond the limits of the pipe before the valve was fully closed. Of course, the full pressure due to the closure was not developed. To obtain this pressure the pipe should be so long that the valve is fully closed before the commencement of the pulsation reaches the head of the pipe and returns. The inquiry was further complicated by using a series of pipes of different diameters running from $1\frac{1}{2}$ ins. up to 6 ins., the longest being 75 ft.

In Professor Carpenter's experiments, first series, there was a 2-in. pipe 30 ft. long containing the valve, then 33 ft. of $2\frac{1}{2}$-in. pipe, then 150 ft. of 3-in., then 375 ft. of 6-in. In the second series there was a $1\frac{1}{2}$-in. pipe $53\frac{1}{2}$ ft. long leading from a tank. The valve was thought to occupy 0.023 second in closing, an interval long enough to allow the pulsation to run 100 ft. In none of these experiments could the full pressure due to closing have been developed. In some of the experiments an air chamber was added to the complications incident to different sizes and inadequate lengths of pipes. In all experimental inquiries, the phenomenon under investigation should be produced as free as possible from the interference and superposition of extraneous phenomena.

The preceding results appear to be all that are necessary for the case of total instantaneous arrest of motion. The closeness with which the formula, with proper modifications, represents the velocity of sound, may be accepted as a guaranty that it is well grounded. In fact, by taking the velocity of the pulsation, in all cases, equal to that of sound, the resulting error would be less than one-tenth in a 5-ft. steel pipe, and in cast-iron pipes of ordinary size the error would be negligible for ordinary purposes.

In this case (total instantaneous arrest of motion) guidance is afforded by these considerations: The flow into the pipe continues with unabated velocity from the instant of stoppage till the arrival of the pulsation at the head of the pipe. This determines the quantity the water above the normal, in the pipe at that instant, and its consequent condition of pressure. For any given pipe no error is made in taking the pressure consequent on the arrest of motion proportional to the velocity.

Save as to very small pipes of short length, the case of sudden and total arrest of motion is not within the range of consideration.

The ordinary case is that of a gradual arrest of motion. Even in this case, the preceding results can give all the aid required for practical purposes. The general method is this : Find the interval t required for a pulsation to traverse the pipe. Then, the diminution of velocity effected during twice this interval may be regarded as a velocity instantaneously arrested, and the pressure deduced from this supposition will be the pressure resulting from the diminution of velocity.

A common question is: "What is the maximum pressure that could result from closing a pipe, in a given time, at a uniform rate ?" The velocity will diminish more rapidly as the movement proceeds, the diminution being most rapid near the close. Suppose the interval $2\,t$ to expire at the close of the movement. Find the position of the gate and consequent velocity at the beginning of that interval. Find the pressure P that would result from the instantaneous arrest of this velocity. A pressure very near P must have existed at the commencement of the interval $2\,t$. Correct the velocity accordingly, and recompute P, etc. By such methods results sufficiently correct for practical purposes can be obtained.

The most important bearing of these principles, as already indicated, is in the regulation of water-wheels supplied by long penstocks. As an instance of the difficulties liable to occur in such applications, consider the power plant of the Pioneer Electric Power Company of Ogden, Utah.*

The length of the pipe line is given as 31 000 ft., a length requiring more than 7 seconds for a pulsation to traverse, and implying a cyclic interval of more than 14 seconds. The pipe is designed for a maximum velocity of 9 ft. per second, the sudden arrest of which would involve, according to what precedes, a pressure above the normal of over 500 lbs. per square inch. The normal pressure in the pipe at the power house is said to be 500 ft. head, or about 217 lbs. per square inch.

In this condition suppose a sudden falling off of one-fifth in the demand for power. By suddenly diminishing the draft of water, and consequently the velocity in the pipe, by one-fifth, the pressure in the pipe is increased by 100 lbs., nearly 50%, and for a few seconds the power would be increased rather than diminished by closing the gates. Consider again the return pulsations, lowering the pressure much below the normal, and some idea is obtained of the difficulties attending the maintenance of a uniform speed under such conditions.

*See *Transactions*, Vol. xxxviii, p. 246.

DISCUSSION.

Mr. Hering. RUDOLPH HERING, M. Am. Soc. C. E.—The assumption upon which the argument of this paper is based is found in the following paragraph: "the diminution of velocity effected during twice" the interval required for a pulsation to traverse the pipe "may be regarded as a velocity instantaneously arrested, and the pressure deduced from this supposition will be the pressure resulting from the diminution of velocity."

It would be well if the author had stated his reasons for this assumption more fully. He leaves the reader in some doubt, and yet this assumption is really the foundation of the paper.

It would also be of interest, if, in addition to a reference to the plant of the Pioneer Electric Power Company, of Ogden, Utah, a comparison of the results of the author's formula had been made with the experiments of Mr. Weston and Professor Carpenter which he criticises In view of the fact that opportunities for properly conducting such experiments are rare, the making of even imperfect sets of experiments should rather be encouraged. Imperfect or incomplete conditions are sometimes the only ones available, and must be taken in order to secure any results whatever. A number of such incomplete experiments, if rightly interpreted, might yet make a sufficiently good basis upon which to support a theory.

Mr. Goldmark. HENRY GOLDMARK, M. Am. Soc. C. E.—The paper under discussion is an interesting attempt to express in mathematical language the increase in pressure resulting from the sudden stoppage of water flowing in a pipe. The fact of such increase, is, of course, well established, though few exact measurements of its magnitude have, as yet, been made. The theoretical values obtained by the author's formulas are higher than those which have generally been used in designing. For this reason it is all the more desirable that their correctness should be practically tested by an extended series of experiments on hydraulic plants in actual operation.

In the meantime, it is clearly necessary to provide for considerable sudden fluctuations in hydraulic pressure, in designing high pressure conduits. It must be remembered, however, that such fluctuations can be reduced to a minimum by the use of proper valves and connections, and more especially by the insertion of a sufficient number of large relief pipes or openings in the main conduit. In designing the plant of the Pioneer Electric Power Company, to which the author refers, especial attention was given to methods of reducing excessive differences of pressure. The plans of the Chief Engineer, C. K. Bannister, M. Am. Soc. C. E., provided for two large relief pipes or shafts near the lower end of the conduit, which allow the water to rise and overflow in case the flow is suddenly shut off below. In

Mr. Goldmark

applying his formulas to this case, the author appears to be in error, as he substitutes in his equation the total length of the conduit from the nozzles in the power house to the reservoir, a distance of 31 000 ft., but does not in any way take these large relief openings into account. They would certainly modify the results obtained very considerably, as they were carefully placed in such a way as to allow pulsations to travel through them almost as freely as through the main conduit. For this reason the 49-in. wooden relief pipe which leaves the 6-ft. steel pipe at its first elbow was with some difficulty so constructed as to make an angle of only 30° with the main conduit.

Notwithstanding these outlets, there will undoubtedly be a decided momentary rise in pressure, with sudden changes in loading. As far as the strength of the lower end of the conduit and its connections are concerned, it is believed that ample safety was secured by increasing the static pressure by 50% in proportioning all details. Even under this pressure the strains in the steel probably nowhere exceed 14 000 to 15 000 lbs. per square inch,—very low stress for the excellent material used.

To obtain a uniform velocity of motion in the machinery, under sudden fluctuations in pressure, dependence is placed on heavy fly wheels and a very efficient combined mechanical and electric governor. It is hoped that this governor, which is exceedingly rapid in its action and has been successfully used in California for some time, will suffice to give a high degree of steadiness to the motion of the machinery. Its previous use in the case of electric light plants makes this expectation appear not unreasonable.

CORRESPONDENCE.

Mr. Noble.

THERON A. NOBLE, M. Am. Soc. C. E.—The writer is very much pleased to see the subject taken up and hopes that it will receive the attention it deserves and will lead to a thorough practical investigation.

It is to be regretted that the author could not have supplemented his analysis with an account of some tests to show that the formulas could be used with safety. Such an investigation following his analysis would be very valuable, and it is to be hoped that some engineer who has the opportunity will find out what actually occurs in a long pipe line of uniform diameter when the water is suddenly shut off. There would seem to be no special difficulty in obtaining such a record.

The writer has found some difficulty in following the author's method of solution, and cannot agree with him in some particulars.

Mr. Noble. To bring these differences out more clearly, the following solution is offered for criticism.

The length of pipe L is taken as indefinite, and afterwards (in determining f) it is taken to be the length the pulsation will travel in one second. It seems to the writer that for a proper analysis L should be taken as the total length of the pipe from the point of entrance to the point of closing. The pressure f would not be the same at all points, but would be 0 at the entrance, and greatest at the point of closing. The compression of the water and expansion of the pipe, being proportional to f, would also be 0 at the entrance and greatest at the point of closing. This does not seem to have been taken into account by the author in his analysis. His formulas (1) and (2) for displacement by compression and expansion are multiplied by f, the greatest pressure at the point of closing, whereas the amount of expansion and compression is determined by the average pressure, which, in a pipe of uniform diameter and cross-section, would be $\frac{1}{2}f$.

The writer bases his solution on the assumption that the energy stored up in the water is equal to the energy expended in the compression of the water and the expansion of the pipe, which would be theoretically correct if both materials were perfectly elastic. The resulting formula for the length of pulsation would be the same as the author's if the average pressure had been taken as f instead of $\frac{1}{2}f$. The formula for f seems to bring nearly the same result as the author's formula, with the same difference due to the assumption of the average pressure.

Let

g =	acceleration due to gravity	=	32.186
W =	weight of 1 cu. ft. of water at 62° Fahr.	=	62.366
M =	modulus of elasticity of steel	=	30 000 000
m =	" " water	=	294 000

R = radius of the pipe in feet.

T = thickness of the metal in the pipe, in inches.

L = total length of pipe in feet.

t = the time consumed in compressing the water and expanding the pipe after the gate is closed.

m' = mass of the moving water.

f = the increase of pressure at the point of closing, in lbs. per sq. in.

v = velocity of water in the pipe just before closing.

l' = the displacement along the axis of the pipe, due to expansion of the metal, in feet.

l'' = the same due to compression of the water.

$l = l' + l''$ = total displacement of water in the pipe, measured along the axis, in feet.

e = elongation of the circumference of the pipe.

e' = increase in diameter of pipe, in feet.

E = total energy in the water at the time of closing the valve, in foot-pounds. Mr. Noble.

p = Strain on the metal, in lbs. per square inch.

$$E = \tfrac{1}{2}\, m'\, v^2 \qquad m' = \frac{W \pi R^2 L}{g}$$

$$E = \frac{W \pi R^2 L v^2}{2g} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

E also = average energy used in expanding and compressing the water times the total displacement.

Average pressure on the area of the pipe $= 144\, \pi\, R^2 \frac{f}{2}$

$$E = 144\, \pi\, R^2 \frac{f}{2}\, l \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

E also $= \frac{1}{2}\, m'\, v^2$, in which $v = \frac{l}{t}$ = velocity of displacement and m' = mass of water in motion.

$$E = \frac{W \pi R^2 L l^2}{2\, g\, t^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

From equations (2) and (3):

$$144\, l \frac{f}{2} = \frac{W L l^2}{2\, g\, t^2}$$

$$\frac{L}{t^2} = \frac{144\, f\, g}{W\, l} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Expansion of the pipe. Determination of l':

$$M = \frac{p}{\left(\frac{l}{2 \pi R}\right)}, \quad p = \frac{2\left(\frac{f}{2}\right) R}{2\, T}, \quad M = \frac{\left(\frac{f R}{2\, T}\right)}{\left(\frac{e}{2 \pi R}\right)} = \frac{\text{stress}}{\text{strain}}$$

$$e = \frac{\pi R^2 f}{2\, M\, T}, \quad e' = \frac{R^2 f}{M\, t} = \text{increase in diameter.}$$

Increase of area in pipe $= \pi \left(R + \frac{e'}{2}\right)^2 - \pi R^2$

$$= \pi \left(R\, e' + \frac{e'^2}{4}\right)$$

e'^2 is so small that it may be neglected. The increase of area would then be $\frac{\pi R^3 f}{M\, T}$. Increase in volume of the pipe $= L\, f\, \frac{\pi R^3}{M\, T}$,

Displacement along the axis due to expansion $= l' = L\, f\, \frac{R}{M\, T}$.

Compression of water in the pipe. Determination of l''.

$$m = \frac{\left(\frac{f}{2}\right) L}{l''} = \frac{\text{stress}}{\text{strain}}$$

$$l'' = \frac{L f}{2\, m} \quad \ldots\ldots\ldots\ldots\ldots\ldots \ldots (5)$$

Mr. Noble.

$$l = l' + l'' = Lf\left(\frac{1}{2m} + \frac{R}{MT}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

From equations 4 and 6:

$$\frac{L}{t^2} = \frac{144\,fg}{W\,L\,f\left(\frac{1}{2\,m} + \frac{R}{MT}\right)}$$

$$\frac{L}{t} = \sqrt{144\frac{g}{W}\left(\frac{1}{\frac{1}{2\,m} + \frac{R}{MT}}\right)} = \sqrt{144\frac{g}{W}\left(\frac{2\,m\,MT}{2\,R\,m + MT}\right)} \quad \ldots (7)$$

Which is the same as the author's formula with the exception of $\sqrt{2}$, due to the difference in assuming the average pressure $= \frac{f}{2}$.

Assuming a 60-in. pipe, $\frac{1}{4}$ in. thick,

$\frac{L}{t} = 6\,044$ ft., which divided by $\sqrt{2} = 4\,274$ ft.

Determination of f:

From (1) and (2) $144\,\pi\,R^2\,l\frac{f}{2} = \frac{W\,\pi\,R^2\,L\,v^2}{2\,g}$

$$144\,l\frac{f}{2} = \frac{W\,L\,v^2}{2\,g}$$

Substituting (6) for l: $144\,L\,\frac{f^2}{2}\left(\frac{1}{2m} + \frac{R}{MT}\right) = \frac{W\,L\,v^2}{2\,g}$

$$f^2 = \frac{v^2\;W}{144\,g\left(\frac{1}{2m} + \frac{R}{MT}\right)}$$

$$f = \frac{v}{12}\sqrt{\frac{W}{g\left(\frac{1}{2m} + \frac{R}{MT}\right)}} = \frac{v}{12}\sqrt{\frac{W}{g}\left(\frac{2\,m\,MT}{2\,m\,R + MT}\right)} \quad \ldots (8)$$

With a 60-in. pipe, $\frac{1}{4}$ in. thick, $v = 4$ ft. per second,

$f = 325.4$, which $\div\ \sqrt{2} = 230$.

By the author's formula $f = 230$.

With a 72-in. pipe, $\frac{9}{16}$ in. thick, $v = 9$ ft. per second.

$f = 760$, which $\div\ \sqrt{2} = 538$.

By the author's formula $f = 560$.

With a 22-in. pipe, $\frac{1}{4}$ in. thick, $v = 4$ ft. per second.

$f = 344$, which $\div\ \sqrt{2} = 243$.

By the author's formula $f = 256$.

The last calculation refers to the conditions prevailing in the Fresno power plant in which so much trouble was experienced with the ordinary relief valve.

It would seem to the writer that this defect in the efficiency of the relief valve, which is caused by the valve closing suddenly after the pressure is relieved, could be wholly remedied by designing the valve

with a slow closing attachment, by which the length of time consumed in closing could be regulated to suit the time required to relieve the energy stored in the water when the valve is open. The surplus energy in the water would then be consumed in friction in passing through the valve instead of compressing the water and expanding the pipe, and thus prevent the continued pulsations. Mr. Noble.

The writer has designed such a valve, which he expects to try in a plant similar to the Pioneer Power Plant to determine its efficiency. Something of the kind is very much needed to protect long pipe lines, particularly in power plants connected with long pipes under pressure, where the flow in the pipe is constantly changing and always liable to be suddenly stopped.

CHARLES W. SHERMAN, Jun. Am. Soc. C. E.—The author's formula (1) is evidently derived as follows: Mr. Sherman.

Stress (hoop tension) in pipe $= \frac{r\,f}{T}$ per unit of area. Increase in circumference then $= \frac{r\,f}{M\,T}$ in one unit, and total increase in circumference $= 2\,\pi\,r\,\frac{r\,f}{M\,T}$.

Then $\Delta\,r = \frac{r^2\,f}{M\,T}$. But this is true only for a homogeneous system of units; *i. e.*, if L and r are in feet, T must also be in feet, and M and m in pounds per square foot.

With this correction in the meaning of the symbols, the author's formulas are correct, except that the 144 should be dropped from formulas (4), (5) and (6), and from the intermediate work.

Then in the case of a 60-in. steel pipe, $\frac{1}{4}$ in. thick, $r = 2.5$, $T = \frac{1}{48}$, $M = 144 \times 30\,000\,000$, $m = 144 \times 294\,000$, $g = 32.16$, $W = 62.5$, and there results $\frac{L}{t} = 2\,549$ ft. per second, instead of 4 272 ft. per second. Neglecting the distension of the pipe, as in formula (6), $\frac{L}{t} = 4\,667$ ft. per second, or practically the same result as obtained by the author.

The writer fails to see that the fact of the velocity of sound in water being practically the same as the figure last obtained has anything to do with the question, either as confirmation or otherwise.

This discussion has considered only the interior pressure of the water on the pipe, and is correct only when there is no pressure, except that of the atmosphere, from without. With the pipe in a trench and covered, the external pressure would be considerable, perhaps enough so that it would be more correct to use the formula which neglects the elasticity of the pipe.

Again, the formulas have been derived on the assumption of a uniform

Mr. Sherman. pressure f throughout the length L, while, as a matter of fact, the pressure is decreasing from a maximum at the piston to zero at a distance L. Assuming that the pressure decreases uniformly, the maximum pressure at the piston would be $2f$, if f represents the average pressure. This, however, does not affect the derivation of the formulas, but it must be borne in mind that the f derived from formula (7) is the average pressure, and that the maximum pressure is $2f$.

By formula (7), if $v = 4$ ft. per second, and $\frac{L}{t} = 2\,549$ ft., $f =$ 138 lbs. per square inch. The maximum pressure $= 2f = 276$ lbs. per square inch.

If, however, the elasticity of the pipe be neglected, the formula becomes

$$f = \frac{v}{L} m$$

With $v = 4$ ft. per second, $L = 4\,667$ ft., there results $f = 252$ lbs. per square inch, and $2f = 504$ lbs. per square inch.

Mr. Le Conte. L. J. Le Conte, M. Am. Soc. C. E.—The subject selected by the author is one which is growing in importance every day, and is very complex. The writer has given much attention to it, both from a theoretic and practical point of view, since December, 1876, when he wrote his first monograph. In the case now under discussion there are several distinct classes of danger to guard against which must not be confounded:

First.—Dangers peculiar to low heads—large volumes of water in motion at moderately high velocities, and moderately heavy iron in the penstock pipe line, naturally of large diameter.

Second.—Dangers peculiar to high heads—small volumes of water under high pressure, very high velocities, and very heavy iron in the penstock pipe line, which is naturally of small diameter.

These are the extreme cases generally met with in practice. In the first case there is ordinarily much more danger to be feared from water-ram than in the second. This arises from the fact that the "shock" has to be sustained by the large pipe—made of thin iron—and the usual thickness given to meet the normal pressures allows a very small margin over and above to resist water-ram.

In the second case there is generally much less to be feared from water-ram, for similar reasons, namely, the penstock is a small pipe of heavy iron, and although the velocity is higher, yet the column of water is smaller, and the heavy iron pipe, with the usual factor of safety, offers a much wider margin to meet water-rams. This advantage in favor of high heads, however, is almost entirely obliterated by another and much greater difficulty encountered in closing the gates under heavy pressure as compared with those manipulated under low pressures.

The general law by which the gate should be closed is: The disc shall be so moved, during the entire time of shutting, that equal quantities of water shall be cut off in equal intervals of time. This means that the column of water shall be stopped or brought to rest by uniformly retarded motion, and at the same time the ram pressure developed shall not exceed a certain safe limit which may be adopted. Mr. Le Conte.

Under very high heads the closing of the gate has no appreciable effect in checking the column of water until the gate is almost closed. The last inch before closing is where the trouble and danger comes in, and where all the time in shutting is expended.

In manipulating the valve there is such a very slight difference between an effect of "no ram pressure" and a "500-lb. ram pressure," that any ordinary mechanism cannot meet the delicate motions which the case requires. The problem is still further complicated by pulsations in the pipe line.

An unusual case came under the writer's observation, where a 30-in. wrought iron pipe line had a gate located at a point some ten miles from the inlet. When it became necessary to close this valve it took three hours of most careful manipulation to shut down the last inch. This valve was soon after removed to a point nearer the inlet.

Gardner S. Williams, Assoc. M. Am. Soc. C. E.—The writer would not venture to submit, in a discussion of so exact and scientific a paper as this, such unscientific and even crude memoranda, were it not that the literature on the subject is extremely limited, and anything which may throw ever so little light upon it seems worthy of consideration. Mr. Williams.

The conditions under which the following results were obtained were in no way ideal, and the observations themselves were made by an inexperienced observer, with only an ordinary Bourdon pressure gauge, as a part of the gate inspection.

The engines running during the observations were two, which were pumping into three 42-in. mains; connecting gates being open and the water free to go through all. Cross lines were connected to these mains supplying adjacent territory at points about 400 ft. apart for their entire length, the diameter of the largest being 12 ins. The system was operating by direct pressure, the standpipe being disconnected from it. The velocity was probably greatest in the main experimented upon, as it was the newest and most direct.

The observations *a*, *b*, *c* and *d* were taken about thirty days apart. The high values for the increase of pressure shown in observation *a* on gates 4 and 5 look suspicious and probably indicate a gauge reading at least 5 lbs. too high. As bearing upon the value of the gauge readings for any purpose except the direct comparison of successive readings, it is to be added that the initial pressures given, when compared with the reading of a Crosby pressure recorder at the pumping station, fail

Mr. Williams. to show a loss of head between the two points that appears consistent with each other or with probable conditions; but whatever error may exist in these readings, with reference to extraneous data, the writer sees no reason to question the value of the observations themselves, except as above noted, in showing the results obtainable with such apparatus under such circumstances. The readings are probably correct within 1 lb. and the time of closing the gate correct within ten seconds. The discs of the gate were operated by a hydraulic cylinder, an index on the stem showing when the gate was opened or closed. The gauge was connected to the up-stream side of the gate by about 2 ft. of ¾-in. pipe, at a point not more than 12 ins. from the discs. The revolution counters on the engines are read hourly, and from these readings the approximate velocity was estimated. The variation of consumption during the time may easily have made the velocity quite different from the estimate at the time of the observation.

TABLE SHOWING OBSERVED EFFECT, ON BOURDON PRESSURE GAUGE IN GATE WELL, OF CLOSING HYDRAULIC GATE ON 42-IN. MAIN AT DETROIT.

Gate number.	Observation.	Time closing.	Initial pressure at gate.	Maximum pressure noted.	Increase of pressure.	Water registered by engine counter during hour in which observation was taken.	Approximate velocity by engine register.	Distance from pumps.	Per cent. increase of pressure.
		M. S.	Lbs.	Lbs.	Lbs.	Gallons.	Feet per second.	Feet.	
3	*a*	11 0	35-41	45	7.0	1 990 800	2.58	3 642	18
	b	10 0	40	45	5.0	1 972 800	2.54	....	12
	c	11 0	37	45	8.0	2 082 920	2.62	....	22
4	*a*	10 30	34-35	50	15.5*	1 954 800	2.52	5 339	45
	b	11 30	37	42	5.0	2 026 800	2.61	....	13
	c	11 0	35	40	5.0	2 099 160	2.70	....	14
5	*a*	11 0	34-36	50	15.0	1 957 320	2.52	6 383	43
	b	11 0	37	40	3.0	1 952 280	2.51	....	9
	c	11 30	33	42	9.0*	2 109 120	2.71	....	27
6	*a*	10 30	30-35	42	9.5	1 815 660	2.34	7 672	30
	b	11 0	35	42	7.0	1 952 280	2.52	....	20
	c	12 0	34	40	6.0	2 151 240	2.77	....	18
	d	13 0	34	40	6.0	2 263 440	2.91	....	18
7	*a*	13 0	33-35	43	9.0	1 805 580	2.32	8 684	27
	b	13 15	37	45	7.0	1 910 880	2.46	....	19
	c	14 0	37	40	3.0	2 151 240	2.77	....	9
	d	14 0	37	40	3.0	2 245 440	2.89	....	9
8	*a*	8 45	32-37	40	5.5	1 867 140	2.43	10 237	16
	b	9 0	35	44	9.0	1 975 320	2.54	....	27
	c	10 0	35	43	8.0*	2 197 560	2.82	....	23
	d	10 0	35	42	7.0	2 273 400	2.92	....	20
9	*a*	12 45	37	45	8.0	1 904 580	2.46	11 991	22
	b	12 50	40	45	5.0	1 975 320	2.55	....	12
	c	12 0	39	45	6.0	2 310 120	2.97	....	16
	d	14 0	36	44	8.0	2 291 400	2.95	....	22

* Recording gauge at pumping station showed a rise of 0.5 to 1.0 lb. at this time.

J. P. Frizell, M. Am. Soc. C. E.—The author finds, in reviewing the paper, that he has stated some points too elliptically and with too little elaboration; forgetting that words do not carry the same force to the minds of others as to his own. As to the difficulty found by Mr. Hering, let this be said: Consider water moving in a pipe and a gate closing to arrest the motion. Counting from any given instant, each infinitesimal diminution of velocity carries with it an infinitesimal increment of pressure. These increments accumulate till the first has had time to run to the head of the pipe and back. At that instant, the aggregate diminution of velocity is that due to the time $2t$ and the increase of pressure is the same as if this diminution had taken place instantaneously. Mr. Frizell.

The difficulty in using the experiments of Mr. Weston and Professor Carpenter is explained in the paper.

As to Mr. Noble's view that the pressure f should be 0 at the entrance to the pipe, this appears to the author to be a misconception. Referring to the diagram, Fig. 1, suppose the water to be moving from C toward A, and $A B$ to be a gate instantaneously closed. Let $C D$ be the section in which the water is in the act of coming to rest. Between $C D$ and $A B$ every particle is at rest; between $C D$ and the origin of the pipe every particle is moving with full velocity. The force producing compression is derived from the loss of velocity. This is obvious as regards the entire mass. It is equally true as regards each individual particle of the mass. Every particle which has come to rest has undergone the full compression due to the act of coming to rest. Therefore all the particles between $A B$ and $C D$ are in the same state of compression and the pressure is uniform.

The author differs from Mr. Sherman in holding that it is a confirmation of the truth of a formula to find it, with suitable modifications, representing well-known physical laws. Mr. Sherman's statement that the author is in error in assuming the pressure uniform is answered above.

The paper attempts to deal with an important engineering problem from a mathematical point of view, and therein differs from the general character of the papers presented to the Society, which more commonly deal with experimental and practical results. By way of excuse for this departure, the author would observe that, while no formula which rests wholly upon theoretical considerations can be used with confidence, it is nevertheless true that a clear comprehension of the theory of a subject is a necessary and indispensable preliminary to any intelligent experiment.

The paper was written with the expectation that it would lead some one possessed of the necessary means to an experimental study of the subject. A municipal engineer in charge of a long line of pipe could readily make the necessary experiments. What is required is: (1) A

Mr. Frizell. slight, but instantaneous, change of velocity in the pipe; (2) a pencil representing, by the amplitude of its movement, the pressure of the water; (3) a band of paper moving isochronously and arranged to receive the mark of the pencil.

For the first, the author suggests a hydrant near a long line of pipe. Apply to it a clack valve, which, on release, will strike dead against its seat and arrest the movement of the water. Means must be adopted for determining the discharge of the valve, in order to ascertain the change of velocity in the main.

For the second element, a steam engine indicator is probably as good as anything.

As to the third element, it will probably be difficult to apply the paper to the indicator barrel in the ordinary manner, for two reasons: (1) Because the barrel would be liable to overflow with water and destroy the paper; (2) the spring attached to the barrel, offering a resistance not uniform, but varying as the amplitude of movement, complicates the imparting of an isochronous movement. For this part of the apparatus, the author suggests attaching the paper to a separate wooden cylinder, larger than the indicator barrel. To this should be attached a motor weight; also a counterweight instead of a spring for keeping the connections taut. To give an isochronous movement to the motor weight in its descent, one obvious method would be to let it act through a shaft carrying vanes or fans. Another would be this: Let the motor weight consist of an accurately turned cylinder of metal, of about 3 ins. diameter. Let it descend in an accurately turned vertical tube, of a little larger diameter, filled with water. It will descend isochronously, and its velocity will depend on the difference in diameter between the tube and the weight.

These experiments need not be attended with any danger of disagreeable results. The valve on the hydrant might, if sufficiently large, bring a severe strain upon the branch leading from the main to the hydrant, but the change of velocity in the main need not be sufficient to bring its safety into question in the remotest degree.